Monotone convergence theorem proof pdf

Given a sequence of $\{f_n\}$ functions that converge point to some limit function $f$$,int \lim_{n\to\infty}f_n = \lim_{n\to\infty}\int f_n.$$ (Take this sequence for example.) Are The Monotonous Convergence Theorem (MCT), The Dominated Convergence Theorem (DCT) and Fatou's Lemma three great results in Lebesgue's integration theory that answer the question When to make $\displaystyle{ \lim_{n\to\infty} }$ and $\int$ commotion? MCT and DCT tell us that if you place certain restrictions on both f_n and f, then you can go ahead and change the limit and in full. Lemma de Fatou, on the other hand, says: Here's the best you can do if you don't make any extra assumptions about the functions. Last week we discussed Lemma de Fatou. Today we will look at an example that uses MCT. And next week we're going to cover the DCT. Monotonous Convergence Theorem: If $\{f_n:X\to[0,\infty)\}$ is a sequence of measurable functions in a measurable set X such as $f_n\to f$ pointwise almost everywhere and $f_1\leq f_2\leq \cdots$, then $$\lim_{n\to\infty}\int_X f_n=\int_X f.$$ Let's look at an example that , on the surface, it looks rather unpleasant. But thanks to MCT, it's not bad at all. ExampleLet X be a measurement space with a positive measure μ and let $f:X\a[0,\infty]$ be a measurable function. Prove that $$\lim_{n\to\infty}\int_X n\log\left(1+\frac{f}{n}\right)d\mu \;\int_X f\;d\mu.$Proof. Start by setting \begin{align*} f_n&=n\log\left(1+\frac{f}{n}\right)\\&\log\left(1+\frac{f}{n}\right)^n \end{align*} and note that each f_n is nonnegative (since both $\log$ and f are nonnegative) and measurable (since the composition of a continuous function with a measurable function is measurable). More $f_1\leq f_2\leq\cdots$. In fact, $\log$ is a growing function, and for a $x\in X$ the left $\string (1+\frac{f(x)}{n}\right)^n$ is increasing. In fact*, it increases to $e^{f(x)}$. In other words, $$\lim_{n\to\infty}f_n(x)=\lim_{n\to\infty}\log\left(1+\frac{f}{n}\right)^n=\log e^{f(x)}=f(x).$$ by the $$\lim_{n\to\infty*\int_X f_n\;d\mu=\int_x f\;d\mu$$$ as desired. It's not so bad, huh? NoticeDid you know that the MCT has a continuous cousin? (Well, maybe it's more like a second cousin.) Have you ever come across Dini's theorem before? Dini's Theorem: If $\{f_n:X\to\mathbb{R}\}$ is a non-rising sequence of continuous functions in a compact metric space X in such a way that $f_n\ to f$ point to a continuous function $f:X\to\mathbb{R}$, then convergence is uniform. Here we have a monotonous sequence of continuous - rather than measurable - functions that converge point to a limit function f in a compact metric space. By Dini's theorem, convergence is uniform. So if f_n are also integrable, then we can conclude** $$\lim_{n\to\infty}\int_Xf_n=\int_Xf.$$ this continuity and measureability analogous (to some extent)! Footnotes *Elementary calculation recall: $\displaystyle{\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n=e^x}$ for any $x\in\mathbb{R}$. ** See Rudin's Principles of Mathematical Analysis (3ed.), Related 7.16.Posts TheoremRead a comment! Given a sequence of $\{f_n\}$ functions that converge point to some limit function $f$$,int \lim_{n\to\infty}f_n = \lim_{n\to\infty}\int f_n.$$ (Take this sequence for example.) Are The Monotonous Convergence Theorem (MCT), The Dominated Convergence Theorem (DCT) and Fatou's Lemma three great results in Lebesgue's integration theory that answer the question When to make $\displaystyle{ \lim_{n\to\infty} }$ and $\int$ commotion? MCT and DCT tell us that if you place certain restrictions on both f_n and f, then you can go ahead and change the limit and in full. 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